

THE WAR OF ATTRITION IN CONTINUOUS TIME WITH COMPLETE INFORMATION*

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1. INTRODUCTION

In this paper, we present a general analysis of the War of Attrition in continuous time with complete information. In this game, each of two players must choose a time at which he plans to concede in the event that the other player has not already conceded. The return to conceding decreases with time, but, at any time, a player earns a higher return if the other concedes first. The game was introduced by Maynard Smith (1974) to study the evolutionary stability of certain patterns of behavior in animal conflicts. It has subsequently been applied by economists to a variety of economic conflicts such as price wars (e.g., Fudenberg and Tirole 1986, Ghemawat and Nalebuff 1985, Kreps and Wilson 1982) and bargaining (Ordover and Rubinstein 1985, Osborne 1985).²

Most authors have assumed specific functional forms for the payoffs. The only general analysis of the equilibria of this game in continuous time with complete information is by Bishop and Cannings (1978). They study a general symmetric game, but restrict the equilibrium analysis to symmetric Nash equilibria which satisfy a certain stability property.³ This paper generalizes their model to allow for asymmetric return functions and arbitrary payoffs in the event that neither player ever concedes. We provide a complete characterization of the Nash equilibrium outcomes.

The paper is organized as follows. In Section 2, we introduce the game and present the assumptions which define the War of Attrition. In Section 3, we state the main theorems and heuristically describe the logic behind the proofs. In Section 4, we relate our analysis to some of the applications which have appeared in the economics literature. Section 5 contains the proofs of the theorems presented in Section 3. Section 6 concludes with a brief discussion of how the analysis changes when time is made discrete.

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² Most of these models incorporate some degree of incomplete information. The method of analysis is similar, but the set of equilibria is sometimes substantially reduced.

³ The equilibrium concept is known as evolutionary stable strategies (ESS). Following Selton (1980), a strategy r is said to be evolutionary stable if (i) r is a best reply to itself and (ii) for any alternative best reply r' to r , r is a better reply to r' than r' is to itself.

2. THE GAME

Two players, a and b , must decide when to make a single move at some time t between 0 and 1.⁴ The payoffs are determined as soon as one player moves. In what follows, α refers to an arbitrary player and β to the other player. If player α moves first at some time t , he is called the *leader* and earns a return of $L_\alpha(t)$. If the other player moves first at time t , then player α is called the *follower* and earns a return of $F_\alpha(t)$. If both players move simultaneously at time t , the return to player α is $S_\alpha(t)$.

In the strategic form of the game, a pure strategy for player α is a time $t_\alpha \in [0, 1]$ at which he plans to move given that neither player moves before that time. Given a strategy pair $(t_a, t_b) \in [0, 1] \times [0, 1]$, the payoff to player α is then defined as follows:

$$\begin{aligned}
P_\alpha(t_a, t_b) &= L_\alpha(t_\alpha) && \text{if } t_\alpha < t_\beta \\
&= S_\alpha(t_\alpha) && \text{if } t_\alpha = t_\beta \\
&= F_\alpha(t_\beta) && \text{if } t_\alpha > t_\beta
\end{aligned}$$

2.1. *Assumptions on the Payoff Functions.* Our first assumption guarantees that the payoff functions are continuous everywhere but on the diagonal.

(A1) L_α and F_α are continuous functions on $[0, 1]$.⁵

Games with this structure are generally called “noisy” games of timing.⁶ Notice that we impose no differentiability conditions on either of these functions. Our next assumption characterizes the War of Attrition.

- (A2) (i) $F_\alpha(t) > L_\alpha(t)$ for $t \in (0, 1)$;
- (ii) $F_\alpha(t) > S_\alpha(t)$ for $t \in [0, 1)$;
- (iii) $L_\alpha(t)$ is strictly decreasing for $t \in [0, 1)$.

Condition (i) requires that the return to following at any time $t > 0$ strictly exceed the return to leading at time t . We do not rule out the possibility that

⁴ This is just a normalization. A game with an infinite horizon can be converted into this framework by a simple change of variable such as $t = z/(1 + z)$, where $z \in [0, \infty)$. See Section 4 on the applications of this model for more discussion on this point.

⁵ It is not essential that $F_\alpha(1)$ and $L_\alpha(1)$ be defined since the only return which can be realized at time 1 is $S_\alpha(1)$. Defining $L_\alpha(t)$ to be continuous at 1 merely allows us to identify $\lim_{t \rightarrow 1} L_\alpha(t)$ with $L_\alpha(1)$. Similarly for F_α .

⁶ The games are called “noisy” because the payoff to the follower depends only on when the other player moves. This reflects the assumption that a player who plans to wait until time t to move does not have to *commit* himself to moving until he has observed the history of the game up to time t . Consequently, if the other player moves before time t , the first player can react optimally, independently of what he had planned to do had the other player not moved at time t . A “silent” game of timing is one in which each player must commit himself to a time at which he will move independently of the action of the other at the outset of the game. Silent games of timing have been also been used in economic models (e.g. Reinganum 1981a, 1981b).

$L_\alpha(0) = F_\alpha(0)$ or the possibility that $F_\alpha(1) = L_\alpha(1)$. Condition (ii), however, requires that the return to following strictly exceed the return to tying at *all* times less than 1. Combined with condition (iii), these conditions imply that, at any time $t < 1$, each player prefers to wait if the other player plans to move but, if forced to move first, would prefer to move sooner than later. Note, however, that, since S_α is not necessarily continuous, our assumptions impose no restrictions on the relation of $S_\alpha(1)$ to either $L_\alpha(1)$ or $F_\alpha(1)$. Consequently, $S_\alpha(1)$ may reflect the equilibrium payoff of *any* continuation game which is played whenever both players wait until time 1.

Although not necessary for our analysis, it will also be convenient to ignore the special case in which return to leading at time 0 is exactly equal to the terminal return.

$$(A3) \qquad L_\alpha(0) \neq S_\alpha(1).$$

In Figure 1, we have illustrated three possible relations between the return functions. In each case, the return functions are normalized so that $S_\alpha(1) = 0$. As required by Assumption A2, in each case the value of F_α lies above L_α over the open interval $(0, 1)$ with L_α strictly decreasing throughout. Figure 1(a) illustrates the case in which both the return to leading and the return to following converge to 0 as time approaches 1. Figure 1(b) illustrates the case where the return from leading is bounded above the return at time 1. Figure 1(c) illustrates the case where the return to leading actually falls below the return at time 1 before time 1 is reached. We will return to each of these examples again in our discussion of the applications. Although not illustrated, Assumption A2 requires that S_α must also lie below F_α over the interval $(0, 1)$. However, it is not necessary that F_α be decreasing.

2.2. *Equilibrium.* It is important for our results to permit agents to randomize across pure strategies. A *mixed strategy* for player α is a probability distribution function G_α on $[0, 1]$.⁷ If we extend the domain of the payoff functions to the set of all pairs of mixed strategies in the obvious way, then a strategy combination (G_α^*, G_β^*) is an *equilibrium* if $P_\alpha(G_\alpha^*, G_\beta^*) \geq P_\alpha(G_\alpha, G_\beta^*)$ for all mixed strategies $G_\alpha, \alpha = a, b$ and $\beta \neq \alpha$.

For the remainder of the paper, (G_a, G_b) will refer to an equilibrium combination, and $q_\alpha(t)$ will denote the probability with which player α moves at exactly time t . We will repeatedly use the fact that if (G_a, G_b) is a pair of equilibrium distributions, then $P_\alpha(t, G_\beta) = \sup_{v \in [0,1]} P_\alpha(v, G_\beta)$ for any t in the support of G_α .

⁷ By a probability distribution on $[t, 1]$, we mean any right-continuous nondecreasing function G from $(-\infty, \infty]$ to $[0, 1]$ with $G(t) = 0$ for $t < 0$ and $G(1) = 1$. Throughout this paper, we will adopt the convention that

$$\int_{t_0}^{t_1} f(v) dG_\alpha(v) = \lim_{t \uparrow t_1} \int_{t_0}^t f(v) dG_\alpha(v).$$

That is, the integral does not include a mass point at time t_1 .

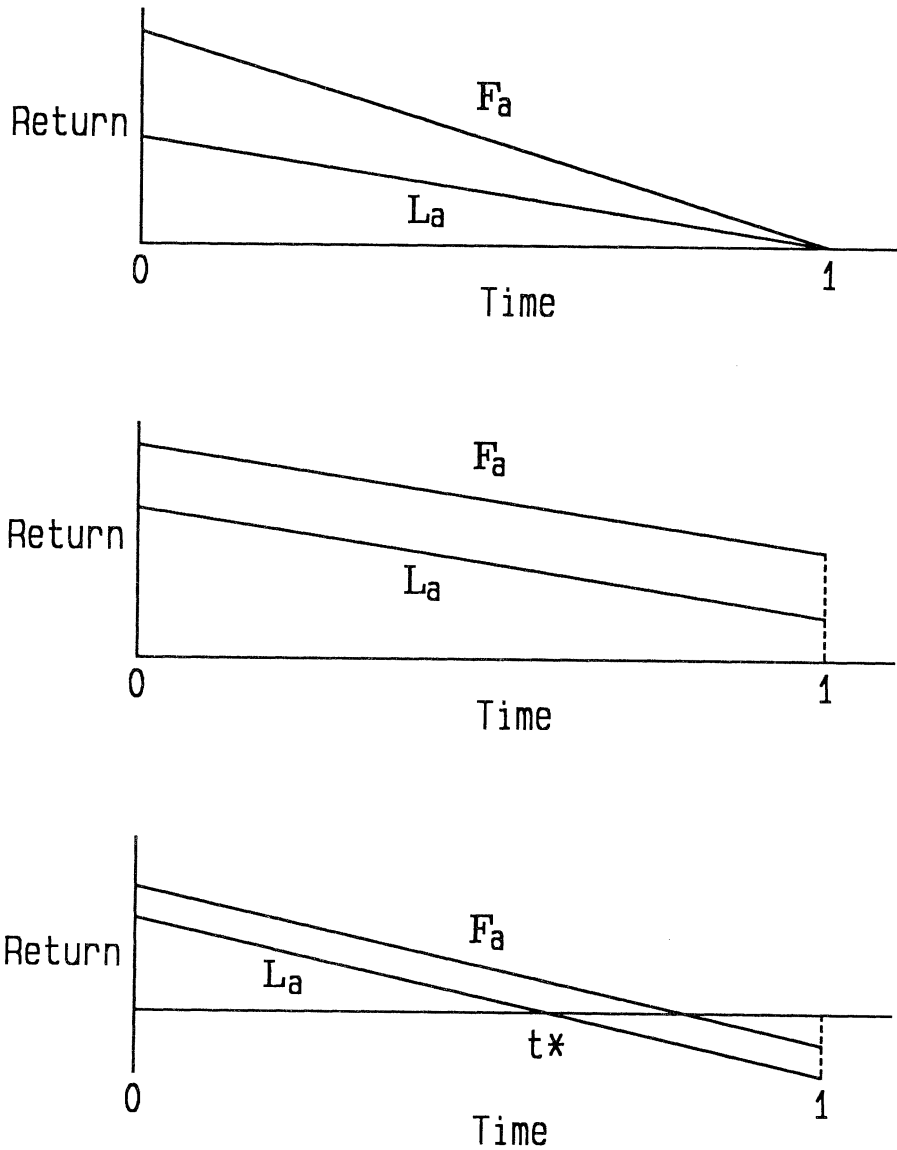


FIGURE 1
THREE PATTERNS FOR THE RETURN FUNCTIONS

3. THE MAIN THEOREMS

In this section we present the main theorems in our analysis and describe the logic behind their proofs. In our analysis, we distinguish between two kinds of equilibria. An equilibrium is *degenerate* if either one of the players moves at time

0 or both wait until the terminal time. There is always at least one equilibrium of this type. In addition, there is sometimes a continuum of *nondegenerate* equilibria. These are equilibria in which both players move over an interval $(0, t)$ according to a strictly increasing, continuous distribution, after which both wait until the terminal time. The indeterminacy comes at the beginning of the game where the only constraint is that only one player may move immediately with positive probability.

3.1. *Degenerate Equilibria.* Theorem 1 characterizes the possible degenerate equilibrium outcomes.

THEOREM 1.

- (a) *There is an equilibrium in which $q_a(1) = q_b(1) = 1$ if and only if $L_\alpha(0) < S_\alpha(1)$ for $\alpha = a, b$.*
- (b) *There is an equilibrium with $q_\alpha(0) = 1$ and $q_\beta(0) = 0$ if and only if, for some α ,*
 - (i) *$L_\alpha(0) \geq S_\alpha(1)$ or*
 - (ii) *for some $t \in (0, 1)$, $L_\alpha(0) \geq F_\alpha(t)$.*

At least one of the equilibrium outcomes described in Theorem 1 always exists. If the return to each player at time 1 is higher than his return from leading at time 0, then there is an equilibrium in which both players wait until time 1 with probability 1. On the other hand, if his return to leading at time 0 exceeds his terminal return, then there is an equilibrium in which player α moves immediately. Moreover, both types of equilibrium may exist if the terminal return of some player α exceeds his return to leading at time 0 which in turn exceeds his return to following at some time later time t . In this case, the equilibrium outcome in which player α moves immediately is supported by a threat by player b to move at time t .⁸

In any case, notice that only one player can move immediately with probability 1. The reason is that Assumption A2 implies that the other player can do better by waiting to obtain the return to following. However, if player α is certain to move immediately with probability 1, then there is generally a large class of equilibrium strategies for player β which support this outcome. The only restriction on his strategy is that it be an optimal response for player α is to move immediately. As we note below, however, the imposition of subgame perfection does eliminate some of this indeterminacy.

3.2. *Nondegenerate Equilibria.* As we demonstrate in Section 4, any mass points in the equilibrium strategies must be concentrated at either the beginning or the end of the game. Furthermore, unless the equilibrium is degenerate, any gaps in the support of the equilibrium distributions must be the same for both players and must extend to the terminal time. The condition that each player be

⁸ As we argue below, however, this equilibrium is not subgame perfect.

indifferent to moving at any time in the interior of the support then defines a pair of integral equations which determine the equilibrium strategies up to some “end point” conditions. The requirements for these endpoint conditions provide the necessary and sufficient for the existence of a nondegenerate equilibrium.

To state these conditions, we require some additional notation. Define

$$I_\beta(s, t) \equiv \exp \left[\int_s^t [dL_\alpha(v)/(F_\alpha(v) - L_\alpha(v))] \right].$$

Note that Assumption (A1) implies that $I_\beta(s, t)$ is well defined everywhere except possibly at 0 and 1. Define $I_\beta(0, 0) \equiv \lim_{t \downarrow 0} I_\beta(0, t)$ and $I_\beta(1, 1) \equiv \lim_{t \uparrow 1} I_\beta(t, 1)$.⁹

THEOREM 2. *There is an equilibrium with $q_a(0) + q_a(1) < 1$ for $\alpha = a, b$ if and only if:*

- (a) $I_a(0, 0) = I_b(0, 0) = 1$; and
- (b) *One of the following conditions is satisfied:*
 - (i) $L_a(t^*) - S_a(1) = L_b(t^*) - S_b(1)$ for some $t^* \in (0, 1]$;
 - (ii) $I_a(1, 1) = I_b(1, 1) = 0$;
 - (iii) $I_x(1, 1) = 0$ and $L_x(1) \geq S_x(1)$ for some x .

Before explaining these conditions, we will proceed with a characterization of the nondegenerate equilibria they imply.

THEOREM 3. *(G_a, G_b) is an equilibrium with $q_a(0) + q_a(1) < 1$ for $\alpha = a, b$ if and only if the conditions of Theorem 2 are satisfied and:*

- (a) $(q_a(0), q_b(0)) \in [0, 1) \times [0, 1)$ and $q_a(0)q_b(0) = 0$.
- (b) *There is a $t_1 \in (0, 1]$ such that, for $\beta = a, b$, either*
 - (i) $L_\beta(t_1) = S_\beta(1)$, or
 - (ii) $t_1 = 1$, and

$$(3.1) \quad \begin{aligned} G_\beta(t) &= 1 - [1 - q_\beta(0)]I_\beta(0, t) \quad \text{for } t \in [0, t_1), \quad \text{and} \\ q_\beta(1) &= [1 - q_\beta(0)]I_\beta(0, t_1) \end{aligned}$$

Consider first the statement of Theorem 3. Condition (a) is an “initial” condition which states that at most one player can move with positive probability at time 0. Since, by Assumption (A2), the return to following exceeds the return to tying at time 0, it always pays at least one player to wait an instant. Condition (b) gives the integral equation which defines the equilibrium strategies after time 0. Both players move according to a continuous increasing probability distribution up to some time t_1 after which they wait until the end of the game. The requirement that player α be willing to move at any point in the interval $(0, t_1)$ implies

⁹ Note that $I_\beta(0, 0)$ and $I_\beta(1, 1)$ can only take on the values 0 and 1. Furthermore, if $F_x(0) > L_x(0)$, then $I_\beta(0, 0) = 1$, and if $F_x(1) > L_x(1)$, then $I_\beta(1, 1) = 1$.

equation (3.1). (If L_α is differentiable, it is the solution to the differential equation, $G'_\beta(t)[1 - G_\beta(t)] = L'_\alpha(t)/[F_\alpha(t) - G_\alpha(t)]$.)

Consider next the statement of Theorem 2. Condition (a) is necessary and sufficient for equation (3.1) to define a strictly increasing function. It will be satisfied whenever the return to following at time 0 strictly exceeds the return to following. Condition (b) gives the three possibilities for the strategies defined by Condition (b) of Theorem 3 to be best responses. Condition (b)(i) corresponds to Condition (b)(i) of Theorem 3. In this case, there is a time at which both players earn returns to leading that equal their terminal returns. Setting t_1 equal to this time then makes both players indifferent between moving at any time in the interval $(0, t_1)$ and waiting until the end of the game.

Conditions (b)(ii) and (b)(iii) correspond to Condition (b)(ii) of Theorem 3. In this case each player β moves according to the distribution function G_β throughout the game. This strategy pair forms a pair of best responses if and only if one of two conditions is satisfied. One possibility is for *both* players to eventually move with probability 1 making the terminal payoff to other irrelevant. The other possibility is for only player α to eventually move with probability one. In this case, the terminal return to player β is irrelevant, but, for the strategy of player α to be optimal, he must not prefer to wait until the end of the game. Consequently, we require that $L_\alpha(1) \geq S_\alpha(1)$. Condition (b) then follows upon observing that equation (3.1) implies that $\lim_{t \uparrow 1} G_\beta(t) = 1$ if and only if $I_\alpha(1, 1) = 0$.

Notice that both Conditions (b)(i) and (b)(ii) might be satisfied simultaneously. In these cases, there are two one parameter families of equilibria which differ according to whether there is a gap in the supports of the equilibrium distributions.

3.3. Subgame Perfection. Many of the applications of the war of attrition arise in situations in which the players cannot *commit* themselves to the time at which they plan to move at the beginning of the game. In these cases, a decision to move at time t is actually a decision to move *first* at time t , given that neither player has already moved by that time. For these games, it may be desirable to refine the equilibrium concept to incorporate the implications of subgame perfection.

To use this concept, however, we must first extend the concept of a strategy to specify the plans of the player upon reaching any time t . As defined in Section 2, the game is in a reduced normal form in that the strategies do not necessarily specify the plans of a player who deviates from his intended action. If the player does not plan to move before time t with probability 1, then we may use Bayes rule to derive the strategy of the player at time t . Otherwise, at every time t for which Bayes does not determine the plan of the player, we must specify a new distribution function stating the strategy of the player for the subgame starting at that time. In the interest of space, we will avoid the details of a precise specification of this larger strategy space and present only heuristic arguments.

Note first that equation (3.1) implies that, for any $t < 1$, there is a positive probability that it will be reached in any nondegenerate equilibrium. It follows

immediately, therefore, that every nondegenerate equilibrium is subgame perfect. For the same reason, any degenerate equilibrium in which both players wait until time 1 is also subgame perfect. Consequently, the only cases in which subgame perfection may be restrictive is when one of the players moves immediately with probability 1. In these cases, subgame perfection not only eliminates some equilibria, but, as Fudenberg et al. and Ghemawat and Nalebuff have shown, it may even eliminate degenerate equilibrium *outcomes* as well.

A general characterization of the subgame perfect equilibrium may be stated as follows.

THEOREM 4. *There is a subgame perfect equilibrium in which $q_\alpha(0) = 1$ if and only if:*

- (a) *either condition (b) of Theorem 2 is satisfied; or*
- (b) (i) $L_\alpha(0) > S_\alpha(1)$; and
- (ii) $L_\alpha(t) < S_\alpha(1)$ implies $L_\beta(t) < S_\beta(1)$ for all $t \in [0, 1)$.

To understand these conditions, we note that there are two ways in which a degenerate outcome might be made subgame perfect. The first is for the players to adopt a nondegenerate equilibrium if the game reaches time $t > 0$. This is possible if and only if condition (b) of Theorem 2 is satisfied. The other possibility is for player α to plan to move with probability 1 upon reaching any time t up to the time t^* where his return to leading equals his terminal return. Thereafter, he waits until the terminal time with probability 1. In response, player β must adopt a strategy which makes this optimal. Here, two conditions must be satisfied. First, player β must not earn a higher return from leading after time t^* than his terminal return. Otherwise, he will move with probability 1 upon reaching any time after t^* which will lead player α to wait at time t^* . Inducting on this argument then leads to an unravelling of the equilibrium. This argument yields condition (b)(ii). Second, since player β earns a lower return from leading after t^* than by waiting until time 1, he will never move after t^* either. Consequently, the return to player α from leading at time t^* must be no less than his terminal return. Assumption (A2) then implies condition (b)(i).

Notice that, in the absence of a nondegenerate equilibrium for any subgame, Theorem 4 implies that it is subgame perfect for some player to move immediately only if and only if $L_\alpha(0) > S_\alpha(1)$ for some player α . Thus, in this case, the two types of degenerate equilibrium outcomes are mutually exclusive. Either one of the players moves with probability 1 at time 0 or both players wait until time 1.

We should point out that in working in continuous time, there are some additional subtleties that do not arise in the discrete time analogues. The problem is that in continuous time a player may move with zero probability at any *point* in an interval, but nevertheless move with positive probability over the entire interval. Consider, for instance, a game in which, upon reaching any period t , the unique subgame perfect outcome is for player α to move immediately with probability 1. Since it cannot be optimal for player β to move at any time at which player α plans to move with probability 1, player β cannot plan to move

with positive probability at any time before the terminal time. If time is discrete, this implies that he waits until the end with probability 1. If time is continuous, however, this restriction only eliminates strategies with mass points. It does not rule out a strictly increasing, continuous distribution function which rises at rate sufficiently low so that player α always prefers to move immediately upon reaching any time t . Consequently, we are left with an infinity of subgame perfect equilibria.

4. NONDEGENERATE EQUILIBRIA IN ECONOMIC APPLICATIONS

In any interesting application of this game, the return to leading at time 0 will exceed his terminal return for at least one of the players. In this case, there is always a subgame perfect degenerate equilibrium in which one of the players moves immediately. The focus of most of the attention in the literature, however, has been on the nondegenerate equilibria.¹⁰ In this section, we discuss the kinds of economic applications for which nondegenerate equilibria are likely to exist.

Ignoring for the moment, the integral condition at time 0, the class of economic applications for which nondegenerate equilibria exist can roughly be divided into those which satisfy Condition (b)(i) of Theorem 2 and those which satisfy Condition (b)(ii). One important class of applications which generally satisfy both conditions are those which require an infinite horizon. Suppose, for instance, that all of the return functions decline at an exponential rate δ . Then, if we transform time according to the formula $z = t/[t + 1]$, simple calculations reveal that not only is $F_\alpha(1) = L_\alpha(1) = S_\alpha(1) = 0$ as in Figure 1(a), but $I_\beta(1, 1) = 0$ as well. In this case, there is a one parameter family of equilibria in which both players move over the entire interval according to a continuous distribution function.

Condition (b)(ii) may also be satisfied for some applications with a finite horizon if the net return to following depends on the amount of time remaining in the game. In their continuous time version of the "chain store" paradox, for example, Kreps and Wilson (1982) assume that a contest for a market between two firms takes place over a predetermined interval $(0, 1)$. They also assume that the benefit from following over leading is proportional to $1-t$ and that the cost of leading (in their case leaving over leading) is proportional to t . Simple calculations again reveal that $I_\beta(1, 1) = 0$. Consequently, under complete information, their model possesses a one parameter family of nondegenerate equilibria in which one of the players is certain to concede by time 1.¹¹

For most economic applications which require a finite horizon, however, the

¹⁰ There are exceptions. Kornhauser, Rubinstein, and Wilson (1987), for example, argue for selecting the degenerate equilibria.

¹¹ This game may also possess another family of nondegenerate equilibria. If the firms were to play a sequence of n such contests (see Fudenberg and Kreps 1985), the terminal payoff to each firm at the end of the k th contest would represent its equilibrium payoff from playing the remaining $n-k$ contests. If this is nonzero and returns are symmetric, the family of nondegenerate equilibria described in condition (i) of Theorem 2 also exists. Notice that in this equilibrium, the behavior of the firms in the k th contest depends on their behavior in subsequent contests through the value of the terminal payoffs.

return to following stays bounded above the return to leading. Consequently, neither Condition (b)(ii) nor (b)(iii) is satisfied, and the only possibility for establishing the existence of an equilibrium is to satisfy Condition (b)(i). There are two reasons why this condition might not be satisfied. The first is that the return to leading at any finite time may strictly exceed the terminal return. For instance, in the oil exploration example studied by Wilson (1983), if the firm has not drilled its lease by a certain time, it loses the lease. In this case, there is no nondegenerate equilibrium because truncating the horizon introduces a downward discontinuity in the payoffs at time 1 as in Figure 1(b). (See Hendricks and Wilson 1985 for a detailed discussion of this issue.)

The second reason nondegenerate equilibria may not exist is that, even in games where the return to leading eventually falls below the terminal return as in Figure 1(c), the return to leading must equal the terminal return at *exactly the same time for both players*. This property is likely to hold only for symmetric games. Consequently, unless there is a special reason to assume that returns are symmetric, such as in the biology models, we should not expect to find a nondegenerate equilibrium. Two economic applications where this result is of interest is in the patent race model of Fudenberg et al. (1983) and the exit model by Ghemawat and Nalebuff (1985). In both of these models, returns are assumed to be asymmetric and, as a result, the nondegenerate equilibria of Theorem 2 are eliminated.

Finally, consider the implications of the integral condition at time 0. In most applications, the return to following strictly exceeds the return to leading at time 0, so that this condition is satisfied. In some cases, however, particularly where the war of attrition is a subgame of a larger game, this condition is less plausible. For example, Hendricks (1987) studies a model of adoption of a new technology in which there are both first and second-mover advantages. Each firm must choose a time at which to adopt. The return functions are continuous and have the property that, initially the returns to leading are increasing, and exceed the returns to following. During this period, first-mover advantages dominate the second-mover advantages, and each firm has an incentive to preempt. Eventually, however, the return to leading decreases, falling below the return to following, so that the second-mover advantage dominates. Consequently, if L_α is decreasing at the point of intersection of F_α and L_α , then Condition (a) of Theorem 2 is not satisfied and hence there are no nondegenerate equilibria for this subgame.

5. THE TECHNICAL ANALYSIS

We begin by establishing the relation between the supports of the strategies of the two players.

LEMMA 1. *Suppose $G_\alpha(t_1) = G_\alpha(t_2) < 1$ for $t_2 > t_1$. Then $G_\beta(t_1) = G_\beta(t_2 + \delta)$ for some $\delta > 0$.*

PROOF. Suppose $G_\alpha(t_1) = G_\alpha(t_2) < 1$ for $t_1 < t_2$ and choose $\varepsilon \in (0, t_2 - t_1)$. Then, for any $t \in (t_1 + \varepsilon, t_2]$, it follows from Assumption A2 that player β prefers

to move at time $t_1 + \varepsilon$ than to move at t since there is no chance that player α will move in the intervening interval:

$$P_\beta(t, G_\alpha) - P_\beta(t_1 + \varepsilon, G_\alpha) = [L_\beta(t) - L_\beta(t_1 + \varepsilon)][1 - G_\alpha(t_1)] < 0.$$

Furthermore, for any $\varepsilon_1 > 0$ sufficiently small, right-continuity of G_α implies that there is an arbitrarily small $\delta > 0$ such that $G_\alpha(t_2 + \delta) - G_\alpha(t_2) < \varepsilon_1$. It then follows from Assumptions A1 and A2 that, for $t \in (t_2, t_2 + \delta)$,

$$\begin{aligned} P_\beta(t, G_\alpha) - P_\beta(t_1 + \varepsilon, G_\alpha) &= \int_{t_1 + \varepsilon}^t [F_\beta(v) - L_\beta(t_1 + \varepsilon)] dG_\alpha(v) + [S_\beta(t) - L_\beta(t_1 + \varepsilon)]q_\alpha(t) \\ &\quad + [L_\beta(t) - L_\beta(t_1 + \varepsilon)][1 - G_\alpha(t)] \\ &= o(\varepsilon_1) + o(\varepsilon_1) + [L_\beta(t) - L_\beta(t_1 + \varepsilon)][1 - G_\alpha(t_2) + o(\varepsilon_1)] < 0. \end{aligned}$$

Letting $\varepsilon \rightarrow 0$, we may then conclude that for any $t \in (t_1, t_2 + \delta]$, there is an earlier time $v \in (t_1, t)$ at which player β prefers to move. Q.E.D.

The next Lemma rules out any mass point after time 0 but before either player moves with probability 1.

LEMMA 2. For $t \in (0, 1)$, $\lim_{v \uparrow t} G_\beta(v) < 1$ implies $q_\alpha(t) = 0$.

PROOF. Suppose, for some $t \in (0, 1)$, that $q_\alpha(t) > 0$. Then, for any $\varepsilon > 0$, there is an (arbitrarily small) $\delta > 0$ such that (i) $L_\beta(t - \delta) - L_\beta(t + \delta) < \varepsilon$, and (ii) $q_\alpha(t + \delta) = 0$ with $G_\alpha(t + \delta) - G_\alpha(t - \delta) < q_\alpha(t) + \varepsilon$. It then follows from Assumptions A1 and A2 that, for ε and δ chosen sufficiently small,

$$\begin{aligned} P_\beta(t + \delta, G_\alpha) - P_\beta(t, G_\alpha) &= [F_\beta(t) - S_\beta(t)]q_\alpha(t) + \int_t^{t + \delta} [F_\beta(s) - L_\beta(t)] dG_\alpha(s) \\ &\quad + [L_\beta(t + \delta) - L_\beta(t)][1 - G_\alpha(t + \delta)] \\ &= [F_\beta(t) - S_\beta(t)]q_\alpha(t) + o(\varepsilon) + o(\varepsilon) > 0. \end{aligned}$$

Similarly, for $v \in (t - \delta, t)$,

$$\begin{aligned} P_\beta(t + \delta, G_\alpha) - P_\beta(v, G_\alpha) &= [F_\beta(v) - S_\beta(v)]q_\alpha(v) + \int_v^{t + \delta} [F_\beta(s) - L_\beta(v)] dG_\alpha(s) \\ &\quad + [L_\beta(t + \delta) - L_\beta(v)][1 - G_\alpha(t + \delta)] \\ &= o(\varepsilon) + [F_\beta(t) - L_\beta(v)]q_\alpha(t) + o(\varepsilon) + o(\varepsilon) > 0. \end{aligned}$$

Consequently, player β will never move in the interval $(t - \delta, t]$. This implies that $G_\alpha(t - \delta) = G_\beta(t)$. Then, if $\lim_{v \uparrow t} G_\beta(v) < 1$, Lemma 1 implies that $G_\alpha(t - \delta) = G_\alpha(t)$, contradicting the hypothesis that $q_\alpha(t) > 0$. Q.E.D.

If G_α is strictly increasing over some interval, then we may use the fact that player α must be indifferent to moving at time within the interval to explicitly

characterize the equilibrium strategy of player β over this interval in terms of the return functions L_α and F_α .

LEMMA 3. Suppose G_α is strictly increasing over the interval $[t_0, t_1]$. Then, for $t_0 > 0$ and $t \in (t_0, t_1)$, $G_\beta(t) < 1$ implies

$$(5.1) \quad G_\beta(t) = 1 - [1 - G_\beta(t_0)]I_\beta(t_0, t).$$

PROOF. Suppose that G_α is strictly increasing over the interval $[t_0, t_1]$. Then, since $G_\alpha(t) < 1$ for $t < t_1$, it follows from Lemma 2 that G_β is continuous on $(0, t_1)$. Therefore, for any $t \in [t_0, t_1)$.

$$(5.2) \quad 0 = P_\alpha(t, G_\beta) - P_\alpha(t_0, G_\beta) \\ = \int_{t_0}^t [F_\alpha(v) - L_\alpha(t_0)] dG_\beta(v) + [1 - G_\beta(t)][L_\alpha(t) - L_\alpha(t_0)]$$

Since G_β and L_α are both monotonic and continuous on $[t_0, t]$, we may apply the formula for integration by parts (Rudin 1964, p. 122) to obtain

$$(5.3) \quad [1 - G_\beta(t)][L_\alpha(t) - L_\alpha(t_0)] \\ = \int_{t_0}^t [1 - G_\beta(v)] dL_\alpha(v) - \int_{t_0}^t [L_\alpha(v) - L_\alpha(t_0)] dG_\beta(v).$$

Substituting (5.3) into (5.2) and rearranging terms then yields, for all $t \in [t_0, t_1)$,

$$(5.4) \quad \int_{t_0}^t [L_\alpha(v) - F_\alpha(v)][1 - G_\beta(v)] \left[\frac{dG_\beta(v)}{1 - G_\beta(v)} - \frac{dL_\alpha(v)}{L_\alpha(v) - F_\alpha(v)} \right] = 0$$

But, since $[L_\alpha(v) - F_\alpha(v)][1 - G_\beta(v)] < 0$ for all $v \in [t_0, t]$, equation (5.4) implies that

$$\int_{t_0}^t dG_\beta(v)/[1 - G_\beta(v)] = - \int_{t_0}^t dL_\alpha(v)/[F_\alpha(v) - L_\alpha(v)].$$

Employing a change of variable (Rudin 1964, p. 122-124), we may then apply the fundamental theorem of calculus to obtain:

$$\log [1 - G_\beta(t)] = \log [1 - G_\beta(t_0)] + \int_{t_0}^t dL_\alpha(v)/[F_\alpha(v) - L_\alpha(v)].$$

Taking antilogs and rearranging terms then yields equation (5.1). Q.E.D.

Note that, if L_α is continuously differentiable, equation (5.1) is simply the solution to the differential equation

$$G'_\beta(t)/[1 - G_\beta(t)] = L'_\alpha(t)/[L_\alpha(t) - F_\alpha(t)]$$

with initial condition $G_\beta(t_0)$. In this case, G_β has a continuous density function g_β over the interval (t_0, t_1) .

We establish next that if neither player moves with probability 1 at time 0, then the game cannot end with certainty until time 1.

LEMMA 4. *Suppose $G_\alpha(0) < 1$. Then $G_\beta(0) < 1$ implies $G_\beta(t) < 1$ for $t < 1$.*

PROOF. Let $\hat{t} = \sup \{t \geq 0: G_\alpha(t) < 1 \text{ for } \alpha = a, b\}$ be the earliest time by which one of the players plans to move with certainty. The lemma is equivalent to the requirement that $\hat{t} \notin (0, 1)$. Suppose $0 < \hat{t} < 1$.

We will show first that the strategy of at least one of the players must have a mass point at time \hat{t} . Suppose not. Then, for some player β , G_β is strictly increasing over an interval (t', \hat{t}) and $\lim_{t \uparrow \hat{t}} G_\beta(t) = 1$. Then, for any $t \in (t', \hat{t})$, Lemma 3 combined with Assumption A2 implies that G_α is strictly increasing over (t', t) . Combining Lemma 3 with Assumption A2 again, we then obtain that $\lim_{t \uparrow \hat{t}} G_\beta(t) = 1 - [1 - G_\beta(t')]I_\beta(t', \hat{t}) < 1$. A contradiction.

But, if $q_\beta(\hat{t}) > 0$, then Lemma 2 implies that $\lim_{t \uparrow \hat{t}} G_\alpha(t) = 1$. The definition of \hat{t} then implies that G_α is strictly increasing over some interval (t', \hat{t}) . It then follows from Assumption A2 and Lemma 3.3 that $\lim_{t \uparrow \hat{t}} G_\alpha(t) < 1$. This contradiction proves the lemma. Q.E.D.

Define

$$t^* \equiv \sup \{t \geq 0: G_\alpha \text{ is strictly increasing on } [0, t) \text{ for } \alpha = a, b\}$$

to be the beginning of the first interval during which one of the players moves with probability 0. Combining Lemmata 1 and 4, we can show that, unless one of the players moves with probability 1 at time 0, neither player ever moves in the interval $(t^*, 1)$.

LEMMA 5. *Suppose $G_\alpha(0) < 1$. Then*

- (i) $G_\beta(t) = 1 - [1 - q_\beta(0)]I_\beta(0, t)$ for $0 \leq t < t^*$, and
- (ii) $G_\beta(t) = G_\beta(t^*)$ for $t^* \leq t < 1$.

PROOF. Suppose that $G_\alpha(0) < 1$. Then Lemma 3 and right-continuity of G_β imply that $G_\beta(t) = 1 - [1 - q_\beta(0)]I_\beta(0, t)$ for $0 \leq t < t^*$. Therefore, the lemma will be proved if we can establish part (ii). Suppose $q_\beta(0) < 1$ and $t^* < 1$. Then Lemma 4 implies that $G_\beta(t^*) < 1$. Let

$$t' = \sup \{v \geq 0: G_\beta(v) = G_\beta(t^*)\}.$$

Since $G_\beta(t^*) < 1$, it follows that $t' \leq 1$. We need to show that $t' = 1$.

Suppose first that $t' = t^*$. Then the definition of t^* implies that, for some $t'' > t'$, $G_\alpha(t'') = G_\alpha(t^*)$. Since $G_\alpha(t^*) < 1$, it then follows from Lemma 1 that $G_\beta(t'') = G_\beta(t^*)$, contradicting our assumption that $t' = t^*$.

Suppose next that $t^* < t' < 1$. Then, since Lemma 4 implies that $G_\alpha(t') < 1$, it follows from Lemma 2 that $G_\beta(t^*) = G_\beta(t') < 1$. But then Lemma 1 implies that, for some $\delta > 0$, $G_\alpha(t^*) = G_\alpha(t' + \delta) < 1$. Applying Lemma 1 again then yields $G_\beta(t^*) = G_\beta(t' + \delta)$, contradicting the definition of t' . Q.E.D.

Lemma 5 implies that the support of the equilibrium strategies is composed of at most an interval $[0, t^*]$ and $\{1\}$. Furthermore, any differences among the equilibrium strategies of player β must occur in the values of either $q_\beta(0)$ or t^* .

5.1. *Equilibrium Restrictions on the Strategies at Time 0.* Next we establish the restrictions imposed on the equilibrium strategies at time 0. They are based on the argument behind Lemma 2 which, at time 0, implies only that at most one player can move with positive probability.

LEMMA 6. $q_a(0)q_b(0) = 0$

PROOF. Suppose that $q_\alpha(0) > 0$. Then, for any $\varepsilon > 0$, there is an (arbitrarily small) $\delta > 0$ such that (i) $L_\beta(0) - L_\beta(\delta) < \varepsilon$, and (ii) $q_\alpha(\delta) = 0$ with $G_\alpha(\delta) - G_\alpha(0) < \varepsilon$. It then follows from Assumptions A1 and A2 that, for ε and δ chosen sufficiently small,

$$\begin{aligned} P_\beta(\delta, G_\alpha) - P_\beta(0, G_\alpha) &= [F_\beta(0) - S_\beta(0)]q_\alpha(0) + \int_0^\delta [F_\beta(s) - L_\beta(0)] dG_\alpha(s) \\ &\quad + [L_\beta(\delta) - L_\beta(0)][1 - G_\alpha(\delta)] \\ &= [F_\beta(0) - S_\beta(0)]q_\alpha(0) + o(\varepsilon) + o(\varepsilon) > 0 \end{aligned}$$

which implies that $q_\beta(0) = 0$.

Q.E.D.

5.2. *Equilibrium Restrictions on the Value of t^* .* In this section, we consider the equilibrium restrictions implied by the value of t^* . These are determined by comparing the payoff to a player from moving at or before time t^* with his payoff from waiting until time 1. Given Assumption A3, there are two cases to consider as determined by the value of t^* .

LEMMA 7. Suppose $t^* = 0$.

- (i) $q_\alpha(0) = 0$ and $q_\beta(0) < 1$ implies $L_\alpha(0) \leq S_\alpha(1)$.
- (ii) $q_\alpha(0) = 1$ implies either $L_\alpha(0) \geq S_\alpha(1)$ or $L_\alpha(0) \geq F_\alpha(t)$ for some $t \in [0, 1)$.

PROOF. Suppose $t^* = 0$.

- (i) If $q_\alpha(0) = 0$, then it follows from Lemma 5 that $q_\alpha(1) = 1$. Therefore, G_α is an optimal response only if

$$0 \leq P_\alpha(1, G_\beta) - \lim_{t \downarrow 0} P_\alpha(t, G_\beta) = q_\beta(1)[S_\alpha(1) - L_\alpha(0)].$$

But if $q_\beta(0) < 1$, then Lemma 5 implies that $q_\beta(1) = 1 - q_\beta(0) > 0$ which in turn implies that $S_\alpha(1) \geq L_\alpha(0)$.

- (ii) If $L_\alpha(0) < S_\alpha(1)$ and $L_\alpha(0) < F_\alpha(t)$ for all $t \in [0, 1)$. Then Assumption A2 implies that

$$\begin{aligned} P_\alpha(1, G_\beta) - P_\alpha(0, G_\alpha) &= [F_\alpha(0) - S_\alpha(0)]q_\beta(0) \\ &\quad + \int_0^1 [F_\alpha(t) - L_\alpha(0)] dG_\beta(t) + [S_\alpha(1) - L_\alpha(0)]q_\beta(1) > 0. \end{aligned}$$

Consequently, $q_\alpha(0) > 0$ cannot be an optimal response.

Q.E.D.

TABLE 1
POSSIBLE INITIAL MASS POINTS WHEN $t^* = 0$

	$L_b(0) > S_b(1)$	$L_b(0) < S_b(1)$
$L_a(0) > S_a(1)$	(1, 0) (0, 1)	(1, 0) (0, 1) ^b
$L_a(0) < S_a(1)$	(0, 1) (1, 0) ^a	(0, 1) ^b (0, 0) (1, 0) ^a

^a If $L_a(0) \geq F_a(t)$ for some $t \in [0, 1)$.

^b If $L_b(0) \geq F_b(t)$ for some $t \in [0, 1)$.

The implications of Lemma 7 are summarized in Table 1. Typical elements represent $(q_a(0), q_b(0))$. The value of x is any number in the interval $[0, 1]$.

When $t^* > 0$, there is an additional restriction to consider besides the tradeoff between moving at time t^* and waiting until time 1. It must also be possible to construct a strategy which makes the other player indifferent between moving at any time near 0. This requires that the “integral” condition at time 0 be satisfied. Recall that $I_\beta(0, 0) \equiv \lim_{t \downarrow 0} I_\beta(0, t)$.

LEMMA 8. *Suppose $0 < t^* < 1$. Then $I_\beta(0, 0) = 1$ and $L_x(t^*) = S_x(1)$.*

PROOF. The requirement that $I_\beta(0, 0) = 1$ follows from Lemma 5 and the requirement that G_x be right-continuous. To establish that $L_x(t^*) = 0$, note that Lemmata 4 and 5 imply that $q_x(1) > 0$. Therefore, G_x is an optimal response only if

$$(5.5) \quad 0 = P_x(1, G_\beta) - \lim_{t \uparrow t^*} P_x(t^*, G_\beta) = [S_x(1) - L_x(t^*)]q_\beta(1)$$

But since Lemmata 4 and 5 also imply that $q_\beta(1) > 0$, it follows that $L_x(t^*) = 0$.

Q.E.D.

Recall that $I_\beta(1, 1) \equiv \lim_{t \uparrow 1} I_\beta(t, 1)$. If $t^* = 1$, and $I_\beta(1, 1) = 0$, then Lemma 5 implies that player β moves with probability 1 before time 1. In this case, the value of $S_x(1)$ is irrelevant. Consequently, when $t^* = 1$, there are no additional restrictions at time t^* unless $I_\beta(1, 1) > 0$ for some player β .

LEMMA 9. *Suppose $t^* = 1$. Then*

- (i) $I_\beta(0, 0) = 1$ and
- (ii) $I_\beta(1, 1) > 0$ implies $L_x(1) \geq S_x(1)$ and $I_x(1, 1)[L_x(1) - S_x(1)] = 0$.

PROOF. The proof of part (i) follows again from Lemma 5 and the requirement that G_β be right-continuous. To establish (ii), note that G_x is an optimal response only if

$$(5.6) \quad 0 \geq P_x(1, G_\beta) - \lim_{t \uparrow 1} P_x(t, G_\beta) = [S_x(1) - L_x(1)]q_\beta(1)$$

Lemma 5 implies that $q_\beta(1) > 0$ whenever $I_\beta(1, 1) > 0$. Therefore, (5.6) implies that

$L_\alpha(1) \geq S_\alpha(1)$. Furthermore, if $I_\alpha(1, 1) > 0$, then it again follows from Lemma 5 that $q_\alpha(1) > 0$, in which case equation (5.5) must be satisfied for $t^* = 1$. Q.E.D.

5.3. *Proof of Theorems.* Using the restrictions derived in Lemmata 1 to 9, we can now prove the theorems stated in Sections 3.1 and 3.2.

PROOF OF THEOREM 1. The necessity of these conditions follow from Lemmata 1 and 7. All that remains is to show that they are sufficient.

(i) If $L_\alpha(0) \geq F_\alpha(t)$ for some $t \in [0, 1)$, then choose the strategies so that $q_\alpha(0) = 1$ and $q_\alpha(t) = 1$. Then, for any $v \in (0, 1]$,

$$P_\beta(v, G_\alpha) - P_\beta(0, G_\alpha) = [F_\beta(0) - S_\beta(0)] > 0$$

which implies that G_β is an optimal response. And

$$P_\alpha(j, G_\beta) - P_\alpha(0, G_\beta) = \begin{cases} [L_\alpha(j) - L_\alpha(0)] \leq 0 & \text{for } j < t \\ [S_\alpha(t) - L_\alpha(0)] < [F_\alpha(t) - L_\alpha(0)] \leq 0 & \text{for } j = t \\ [F_\alpha(t) - L_\alpha(0)] \leq 0 & \text{for } j > t \end{cases}$$

which implies that G_α is an optimal response.

If $L_\alpha(0) \geq S_\alpha(1)$, then a similar argument establishes that $q_\alpha(0) = 1$ and $q_\beta(t) = 1$ form a pair of best responses.

(ii) If $L_\alpha(0) \leq S_\alpha(1)$ and $q_\alpha(1) = 1$ for $\alpha = a, b$, then, for any $t < 1$,

$$P_\beta(1, G_\alpha) - P_\beta(t, G_\alpha) = S_\beta(1) - L_\beta(t) \geq 0$$

which implies that $q_\beta(1) = 1$ is an optimal response.

Q.E.D.

PROOF OF THEOREM 2. The necessity of Condition (a) follows from Lemmata 8(i) and 9(i). The necessity of Condition (b) follows from Parts (ii) of Lemmata 8 and 9. The sufficiency of these conditions then follows upon verifying that one of the strategy pairs defined in Theorem 3 are equilibria. Q.E.D.

PROOF OF THEOREM 3. The necessity of Condition (a) follows from Lemma 6. The necessity of Condition (b) follows from Parts (ii) of Lemmata 8 and 9. The sufficiency of these conditions then follow upon verifying that one of the strategy pairs defined in Theorem 3 are equilibria. Q.E.D.

6. DISCRETE VERSUS CONTINUOUS TIME

Hendricks and Wilson (1985a, 1985b) have studied the equilibrium properties of Wars of Attrition in discrete time and investigated the relation between the equilibria of these games to the equilibria of the continuous time analogues. In this paper, we confine ourselves to some remarks about the differences between the two formulations as they pertain to our characterization theorems.

In both formulations, the same *degenerate* equilibrium outcomes obtain under roughly the same conditions, although as noted in Section 3.3, the number of

subgame perfect equilibria tends to be much larger when time is continuous. Moreover, when the horizon is infinite, the class of nondegenerate equilibria in the continuous time model essentially coincides with the discrete time model. In both cases, there is an equilibrium corresponding to any initial condition in which only one of the players moves with positive probability at time 0.

When the horizon is finite, however, the set of nondegenerate equilibria may diverge substantially. First, in discrete time, there is generally at most one nondegenerate equilibrium, corresponding to the continuous time equilibrium in which neither player moves with positive probability at time 0. In contrast, when time is continuous, the existence of a single nondegenerate equilibrium implies the existence of a continuum of nondegenerate equilibria.¹²

Second, there is always an equilibrium in discrete time whenever the terminal return lies below the return to leading at any time for both players. In contrast, there is no degenerate equilibrium in continuous time when the terminal return lies strictly below the return to leading as time approaches the terminal date. This failure of upper semi-continuity in the equilibrium correspondence is the result of the fact that the payoffs are not continuous in the norm topology on the space of distribution strategies.

These differences in the set of equilibria are reflected in the rich local structure of the equilibria in discrete time games which is totally lacking in the continuous time analogues. This local structure is particularly sensitive to the treatment of ties, which, in continuous time, occur with probability 0. We also note that there is no analogue to the integral conditions at time 0 in the discrete time game.

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